Problem Set on Curvilinear Coordinates:

[Q.1] Express Curl $\vec{A} = \nabla \times \vec{A}$ in orthogonal curvilinear coordinates.

Hint! $\nabla \cdot \vec{A} = A_1 \hat{e}_1 + A_2 \hat{e}_2 + A_3 \hat{e}_3$

$\nabla \times (A_1 \hat{e}_1) = \nabla \times (A_1 h_1 \nabla u_1) = \nabla (A_1 h_1) \times \nabla u_1 + A_1 h_1 \nabla \times \nabla u_1$ (curl(grad)=0)

$\nabla \times (A_1 \hat{e}_1) = \nabla \left(A_1 h_1 \right) \times \frac{\hat{e}_1}{h_1}$

$= \left[\frac{\hat{e}_1}{h_1} \frac{\partial}{\partial u_1} (A_1 h_2) + \frac{\hat{e}_2}{h_2} \frac{\partial}{\partial u_2} (A_1 h_1) + \frac{\hat{e}_3}{h_3} \frac{\partial}{\partial u_3} (A_1 h_1) \right] \times \frac{\hat{e}_1}{h_1}$

$= \frac{\hat{e}_2}{h_3 h_1} \frac{\partial}{\partial u_3} (A_1 h_1) - \frac{\hat{e}_3}{h_1 h_2} \frac{\partial}{\partial u_2} (A_1 h_1)$.

[Q.2] Express $\nabla^2 \psi$ (Laplacian) in orthogonal curvilinear coordinates

Hint: $\nabla^2 \psi \equiv \nabla \cdot \nabla \psi$ { You know $\nabla \cdot \vec{A}$ and $\nabla \psi$ expression from previous notes

~~[Q.3] Express in cylindrical coordi~~

[Q.3] Express $\nabla \phi$ in cylindrical coordinates.

[Q.4] Express $\nabla \phi$ in spherical coordinates.

[Q.5] Prove that a cylindrical coordinate system is orthogonal.

[Q.6] Find the scale factors in elliptic cylindrical coordinates.